

## REPORT DOCUMENTATION PAGE

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## Report Title

Using the Maximum Entropy Principle as a Unifying Theory for Characterization and Sampling of Multi-scaling Processes in Hydrometeorology

## ABSTRACT

This document summarizes the first year work of the project during 21 June 2010 to 31 July 2011 conducted at University of California at Irvine (UCI) before the PIs moved to Georgia Institute of Technology (GaTech). This final report is a replica of the technical report as part of the first annual report submitted earlier.

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**Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:**

**(a) Papers published in peer-reviewed journals (N/A for none)**

Received      Paper

**TOTAL:**

**Number of Papers published in peer-reviewed journals:**

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**(b) Papers published in non-peer-reviewed journals (N/A for none)**

Received      Paper

**TOTAL:**

**Number of Papers published in non peer-reviewed journals:**

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**(c) Presentations**

1. Wang, J, and R. L. Bras, An MEP model of surface heat fluxes, 8th Annual Meeting of Asia Oceania Geosciences Society, Taipei, Taiwan, 8-12 August 2011.
2. Nieves, V., J. Wang, and R. L. Bras, A Bayesian analysis of scale-invariant processes, 31st International Workshop on Bayesian Inference and Maximum Entropy Methods in Science and Engineering, Waterloo, Canada, 10-15 July 2011.
3. Wang, J, and R. L. Bras, An application of maximum entropy production principle in modeling heat fluxes over land surface, 31st International Workshop on Bayesian Inference and Maximum Entropy Methods in Science and Engineering, Waterloo, Canada, 10-15 July 2011.
4. Wang, J, and R. L. Bras, On ignorance priors and the principle of indifference, 31st International Workshop on Bayesian Inference and Maximum Entropy Methods in Science and Engineering, Waterloo, Canada, 10-15 July 2011.
5. Bras, R. L., and J. Wang, An MEP model for remote sensing of evapotranspiration (invited talk), AGU Fall Meeting, San Francisco, 2010.
6. Nieves, V., E. Wood, J. Wang, and R. L. Bras, Maximum entropy distributions of scale-invariant processes, AGU Fall Meeting, San Francisco, 2010.
7. Wang, J., R. L. Bras and V. Nieves, Use of the entropy methods in modeling eco-hydro-geomorphological processes (invited talk), AGU Fall Meeting, San Francisco, 2010.

Number of Presentations: 7.00

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**Non Peer-Reviewed Conference Proceeding publications (other than abstracts):**

Received Paper

**TOTAL:**

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

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**Peer-Reviewed Conference Proceeding publications (other than abstracts):**

Received Paper

**TOTAL:**

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

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**(d) Manuscripts**

Received Paper

**TOTAL:**

Number of Manuscripts:

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**Books**

Received Paper

**TOTAL:**

**Patents Submitted**

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**Patents Awarded**

---

**Awards**

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**Graduate Students**

NAME

PERCENT SUPPORTED

**FTE Equivalent:**

**Total Number:**

**Names of Post Doctorates**

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
Veronica Nieves	1.00
<b>FTE Equivalent:</b>	<b>1.00</b>
<b>Total Number:</b>	<b>1</b>

#### Names of Faculty Supported

<u>NAME</u>	<u>PERCENT SUPPORTED</u>	National Academy Member
Rafael Bras	0.00	Yes
<b>FTE Equivalent:</b>	<b>0.00</b>	
<b>Total Number:</b>	<b>1</b>	

#### Names of Under Graduate students supported

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
<b>FTE Equivalent:</b>	
<b>Total Number:</b>	

#### Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: ..... 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:..... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:..... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):..... 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:..... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense ..... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:..... 0.00

#### Names of Personnel receiving masters degrees

NAME

**Total Number:**

#### Names of personnel receiving PHDs

NAME

**Total Number:**

#### Names of other research staff

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
Jingfeng Wang	1.00
<b>FTE Equivalent:</b>	<b>1.00</b>
<b>Total Number:</b>	<b>1</b>

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**Sub Contractors (DD882)**

**Inventions (DD882)**

**Scientific Progress**

The project during the first year was focusing on (1) deriving the maximum entropy (MaxEnt) distributions of Type I and II multi-scaling processes and establish the links between the multi-scaling distributions and the aggregated properties of the corresponding field variables; (2) developing a model of evapotranspiration (ET) over the land surfaces using the Principle of Maximum Entropy Production (MEP).

## 1. Derivation and validation of MaxEnt distributions of Type I multi-scaling processes

Following the MaxEnt formalism, the probability distribution of a Type I multi-scaling process (i.e. self-similar process with constant parameters),  $z$ , has been derived under the constraints of given multi-scaling moments and geometric mean of the incremental process  $|z_1 - z_2|$ ,

where  $Z$  is the partition function (normalization factor),  $\gamma_0$  is determined from the constraint of given geometric mean,  $M$  the highest order of multi-scaling moment of the incremental process and  $\gamma_q$  the Lagrangian multipliers corresponding to the given multi-scaling moments. Figure 1 The MaxEnt distributions have been validated against empirical histograms of soil moisture and topographic fields (not shown). The findings have been published in Physical Review Letters.

## 2. Derivation and validation of MaxEnt distributions of Type II multi-scaling processes

The probability distribution of the more general case of Type II multi-scaling (i.e. multi-fractal) process, similar to that of Type I, where the parameters are described by probability distributions has also been derived following the MaxEnt formalism where an additional constraint of multi-fractal condition to those of multi-scaling moments and geometric mean was imposed. The MaxEnt distributions have been validated against empirical histograms of topographical fields. The findings has been published in Geophysical Review Letters.

function in terms of the surface fluxes (latent, sensible and ground); and (3) solve numerically the heat fluxes as functions of input variables of net radiation, temperature and humidity at/near the surface. A key component of the MEP model is the concept of "thermal inertia" for latent heat flux, which was postulated according to three heuristic arguments: (1) the turbulent mixing responsible for the transport of heat in the ABL is also responsible for the transport of water vapor, (2) evaporation/transpiration may be expressed in terms of surface soil/leaf surface temperature and humidity according to the maximum principle of evaporation so that the thermal inertia should be expressed in terms of these two surface variables as well, and (3) water vapor within an infinitesimal layer next to the evaporating (soil/leaf) surface is presumably in equilibrium with the liquid water within the soil/leaf-tissues. The model for the case of bare soil is expressed as follows, with

where  $R_n$  is the net radiation,  $T_s$  the surface temperature,  $q_s$  the surface specific humidity,  $I_s$  the thermal inertia of the soil,  $I_0$  is the "apparent thermal inertia of the air"  $\gamma$  is the latent heat of vaporization of liquid water,  $C_p$  the heat capacity of air at constant pressure, and  $R_v$  the gas constant of water vapor.  $E$ ,  $H$  and  $G$  can be solved from the three nonlinear algebraic equations for given input of  $R_n$ ,  $T_s$ , and  $q_s$ , referred to as the MEP model of ET over non-vegetated land surfaces. Figure 2 shows a test of the MEP model using field observations.

The MEP model of ET over vegetated surfaces can be obtained through setting  $G=0$  in the above equations,

where all variables are the same as those defined for the case of non-vegetated surfaces. Figure 3 shows an example of the MEP model predicted vs observed  $E$  and  $H$ .

The findings have been published in Water Resources Research (see below), which was one of the most popular papers (in terms of number of downloads).

## Technology Transfer

# Final Report of ARO W911NF-10-1-0236 (UCI)

## Using the Maximum Entropy Principle as a Unifying Theory for Characterization and Sampling of Multi-scaling Processes in Hydrometeorology

Co-PIs: Rafael L. Bras and Jingfeng Wang  
with Dr. Veronica Nieves

25 January 2012

This document summarizes the first year work of the project during 21 June 2010 to 31 July 2011 conducted at University of California at Irvine (UCI) before the PIs moved to Georgia Institute of Technology (GaTech). This final report is a replica of the technical report as part of the first annual report submitted earlier.

### Activities and Findings

The project during the first year was focusing on (1) deriving the maximum entropy (MaxEnt) distributions of Type I and II multi-scaling processes and establish the links between the multi-scaling distributions and the aggregated properties of the corresponding field variables; (2) developing a model of evapotranspiration (ET) over the land surfaces using the Principle of Maximum Entropy Production (MEP).

#### 1. Derivation and validation of MaxEnt distributions of Type I multi-scaling processes

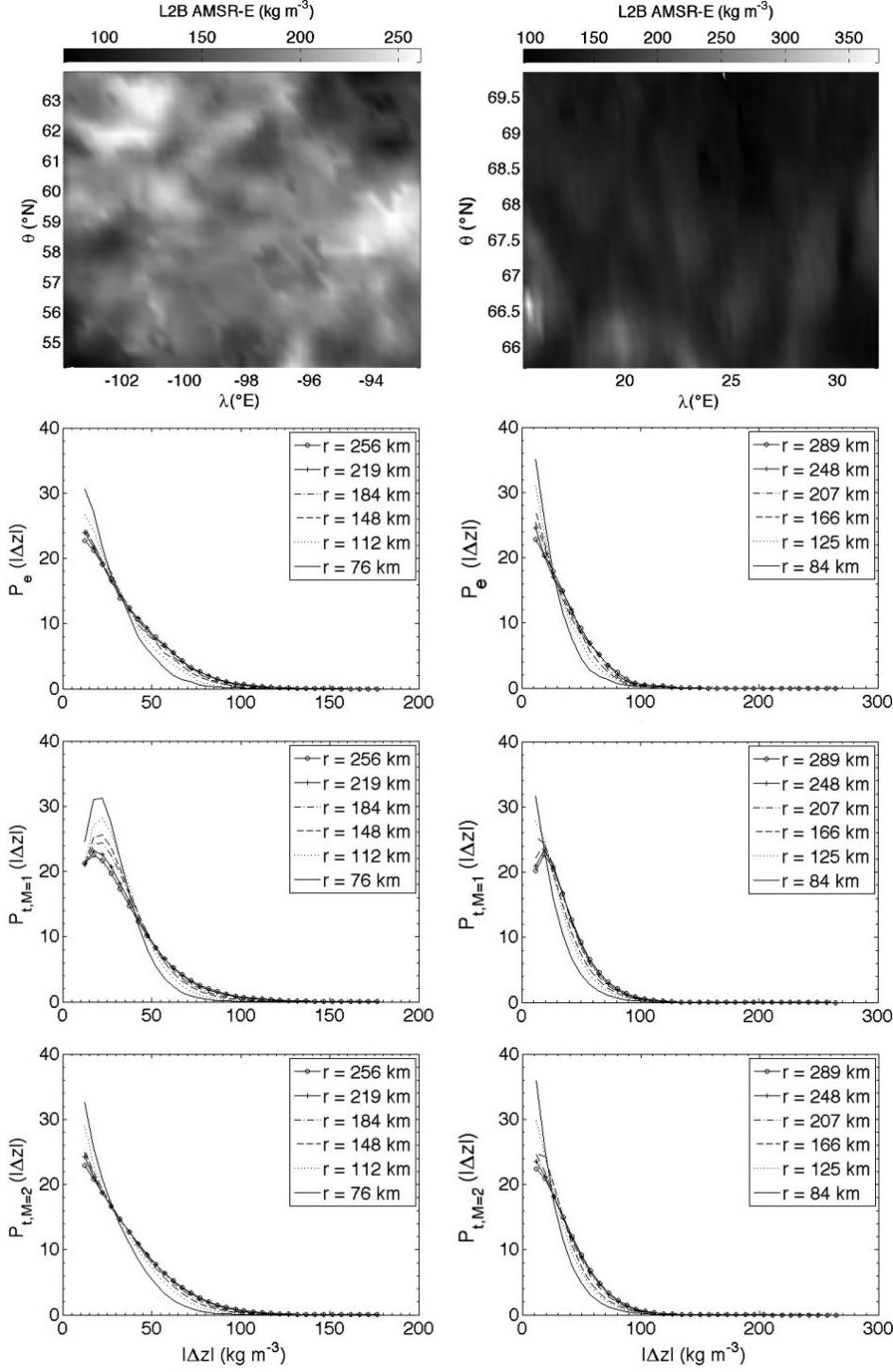
Following the MaxEnt formalism, the probability distribution of a Type I multi-scaling process (i.e. self-similar process with constant parameters),  $z$ , has been derived under the constraints of given multi-scaling moments and geometric mean of the incremental process  $|z_1 - z_2|$ ,

$$p(z_1, z_2) = \frac{1}{Z} |z_1 - z_2|^{-\mu_0} \exp\left(\sum_{q=1}^M \mu_q |z_1 - z_2|^q\right),$$

where  $Z$  is the partition function (normalization factor),  $\mu_0$  is determined from the constraint of given geometric mean,  $M$  the highest order of multi-scaling moment of the incremental process and  $\mu_q$  the Lagrangian multipliers corresponding to the given multi-scaling moments. **Figure 1** The MaxEnt distributions have been validated against empirical histograms of soil moisture and topographic fields (not shown). The findings have been published in *Physical Review Letters*.

#### 2. Derivation and validation of MaxEnt distributions of Type II multi-scaling processes

The probability distribution of the more general case of Type II multi-scaling (i.e. multi-fractal) process, similar to that of Type I, where the parameters are described by probability distributions has also been derived following the MaxEnt formalism where an additional constraint of multi-fractal condition to those of multi-scaling moments and geometric mean was imposed. The MaxEnt distributions have been validated against empirical histograms of topographical fields. The findings will be published in *Geophysical Review Letters* (in press).



**Figure 1.** From top to bottom. Left panels: L2B AMSR-E soil moisture map for October 18, 2009 and region R1SSM, associated empirical ( $p_e$ ) and the MaxEnt distributions ( $p_t$ ) for  $M=1,2$  ( $p_{t;M=1}$  and  $p_{t;M=2}$ ). Right panels: same for region R2SSM. Maps are represented in longitude and latitude. All probabilities are plotted versus the absolute value of the increments  $\Delta z = |z(x_1) - z(x_2)|$  for different separation distances  $r = |x_1 - x_2|$  where  $x_1$  and  $x_2$  are two locations over a two-dimensional domain.

### 3. Development and test of a MEP model of ET over the land surfaces

Built on the case of dry soil, a MEP model of ET has been formulated following the MEP formalism. The formulation has three steps: (1) formulate the dissipation function including latent heat flux (evaporation/transpiration) term; (2) find the stationary point of the dissipation

function in terms of the surface fluxes (latent, sensible and ground); and (3) solve numerically the heat fluxes as functions of input variables of net radiation, temperature and humidity at/near the surface. A key component of the MEP model is the concept of “thermal inertia” for latent heat flux, which was postulated according to three heuristic arguments: (1) the turbulent mixing responsible for the transport of heat in the ABL is also responsible for the transport of water vapor, (2) evaporation/transpiration may be expressed in terms of surface soil/leaf surface temperature and humidity according to the maximum principle of evaporation so that the thermal inertia should be expressed in terms of these two surface variables as well, and (3) water vapor within an infinitesimal layer next to the evaporating (soil/leaf) surface is presumably in equilibrium with the liquid water within the soil/leaf-tissues. The model for the case of bare soil is expressed as follows,

$$G = \frac{B(\sigma)}{\sigma} \frac{I_s}{I_0} H |H|^{-\frac{1}{6}}$$

$$B(\sigma) = 6 \left( \sqrt{1 + \frac{11}{36} \sigma} - 1 \right),$$

$$E = B(\sigma) H$$

$$R_n = E + H + G, \quad \text{with} \quad \sigma = \frac{\lambda^2}{C_p R_v} \frac{q_s}{T_s^2},$$

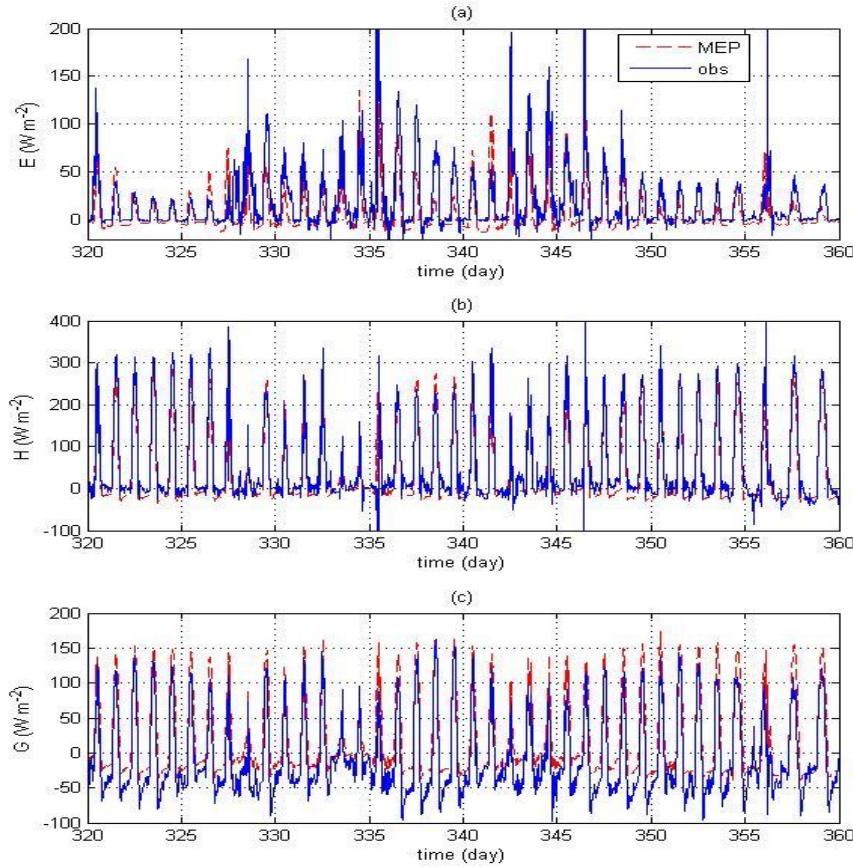
where  $R_n$  is the net radiation,  $T_s$  the surface temperature,  $q_s$  the surface specific humidity,  $I_s$  the thermal inertia of the soil,  $I_0$  is the “apparent thermal inertia of the air”  $\lambda$  is the latent heat of vaporization of liquid water,  $C_p$  the heat capacity of air at constant pressure, and  $R_v$  the gas constant of water vapor.  $E$ ,  $H$  and  $G$  can be solved from the three nonlinear algebraic equations for given input of  $R_n$ ,  $T_s$ , and  $q_s$ , referred to as the MEP model of ET over non-vegetated land surfaces. **Figure 2** shows a test of the MEP model using field observations.

The MEP model of ET over vegetated surfaces can be obtained through setting  $G=0$  in the above equations,

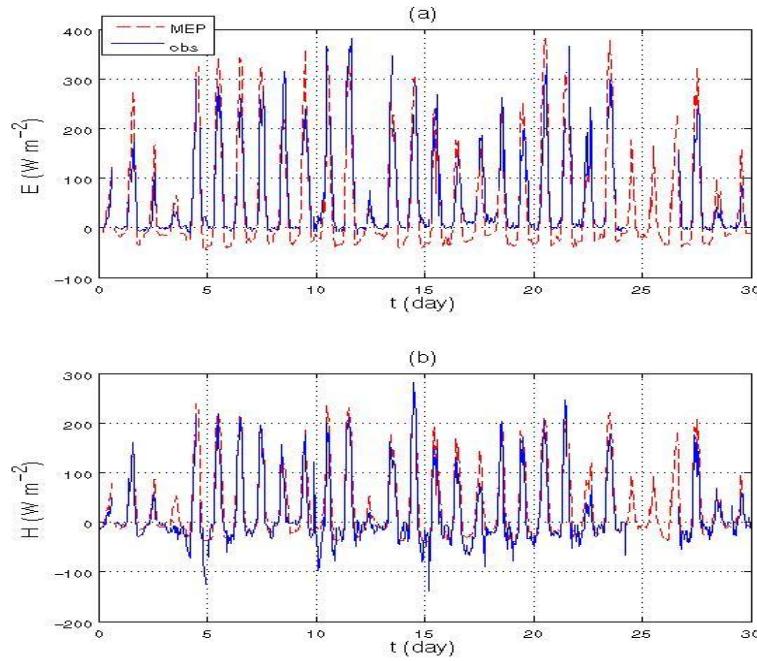
$$E = \frac{R_n}{1 + B^{-1}(\sigma)}, \quad H = \frac{R_n}{1 + B(\sigma)},$$

where all variables are the same as those defined for the case of non-vegetated surfaces. **Figure 3** shows an example of the MEP model predicted vs observed  $E$  and  $H$ .

The findings have been published in *Water Resources Research* (see below), which was one of the most popular papers (in terms of number of downloads).



**Figure 2** Evaporation  $E$ , sensible heat flux  $H$ , and ground heat flux  $G$  predicted by the MEP model (broken red), according to Eq (1), versus the corresponding observed fluxes (solid blue) at Lucky Hills site of the Walnut Gulch Experimental Watershed 16 Nov–26 Dec 2007 [Wang and Bras, 2011]. Three rain events occurred during this period with three wetting and drying cycles of soil moisture (data not shown here, but can be found in [Wang and Bras, 2011]).



**Figure 3** Latent  $E$  and sensible heat flux  $H$  (broken red), predicted by the MEP model using the observed  $q_s$ ,  $T_s$  and  $R_n$  (not shown) versus the observed fluxes (solid blue) at the Harvard Forest (an AmeriFlux site with eddy-covariance flux tower) during 19 August – 8 September 1994 (data courtesy of Steven Wofsy of Harvard University).

## Journal Publications

1. Nieves, V., J. Wang, and R. L. Bras (2011), Statistics of multi-fractal process using the maximum entropy method, *Geophys. Rev. Lett.*, doi:[10.1029/2011GL048716](https://doi.org/10.1029/2011GL048716), in press.
2. Wang, J., and R. L. Bras (2011), A model of evapotranspiration based on the theory of maximum entropy production, *Water Resour. Res.*, 47, W03521, doi:[10.1029/2010WR009392](https://doi.org/10.1029/2010WR009392).
3. Nieves, V., J. Wang, and R. L. Bras (2010), Maximum entropy distributions of scale-invariant processes, *Phys. Rev. Lett.*, 105, 118701, doi:[10.1103/PhysRevLett.105.118701](https://doi.org/10.1103/PhysRevLett.105.118701).

## Conference Presentations

1. Wang, J, and R. L. Bras, An MEP model of surface heat fluxes, *8<sup>th</sup> Annual Meeting of Asia Oceania Geosciences Society*, Taipei, Taiwan, 8-12 August 2011.
2. Nieves, V., J. Wang, and R. L. Bras, A Bayesian analysis of scale-invariant processes, *31<sup>st</sup> International Workshop on Bayesian Inference and Maximum Entropy Methods in Science and Engineering*, Waterloo, Canada, 10-15 July 2011.
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